sAP® Calculus AB Required Summer Work

This summer assignment is intended to be an **independent assignment** to review the prerequisite topics that are needed for AP Calculus. This assignment will also be a useful guide to refer to topics within algebra, geometry, trigonometry, and function analysis. In the first section, you will see a list of prerequisite topics as well as resources where you can review these specific topics. In the following sections, you will complete a table on families of functions as well as complete some problems that focus on prerequisite skills. This assignment is worth **a test grade.**

Prerequisite Topics

Review the table of prerequisite topics. Resources have been provided for each topic if any review or explanation is necessary. This table of topics and resources serves as an excellent primer to the AP Calculus courses.

Algebra Topics	
Topic	Resource
Equation of a line	Linear Equations in Slope-Intercept Form Linear Equations in Point-Slope Form Linear Equations in Standard Form
Rational expressions	Simplifying Rational Expressions
Functions: Domain and Range	Determine Domain of a Function Determine Range of a Function
Functions: Compositions	Find Composite Functions
Functions: Inverses	Introduction to Inverse Functions

Geometry Topics	
Topic	Resource
Area Formula	Area Formulas
Volume and Surface Area Formulas	Volume and Surface Area Formulas
Similar Triangles	Similar Triangles

Trigonometry Topics	
Topic	Resource
Sum and Difference Formulas	Use Sum and Difference Formulas
Double-Angle Formulas	Using Double-Angle Formulas
Trigonometric Identities	Pythagorean Identities Reciprocal and Quotient Identities
Unit Circle	Special Points on the Unit Circle Unit Circle Generating Trigonometric Graphs
Trigonometric Graphs	Graphs of Sine and Cosine Graphs of Tangent and Reciprocal Functions

Functions	
Topic	Resource
Linear Functions	Linear Equations
Polynomial Functions	Solving Polynomials
Rational Functions	Asymptotes of Rational Functions
Exponential Functions	Exponential Function and Its Graph
Logarithmic Functions	Logarithmic Function and Its Graph
Trigonometric Functions	Introduction to Trigonometry
Inverse Trigonometric Functions	Inverse Trigonometric Functions and Their Graphs
Piecewise Functions	Piecewise Functions and Their Graphs Absolute Value Function as a Piecewise Function

Parent Functions

The following represents a list of parent functions you are expected to know **prior** to enrolling in AP Calculus. Complete the following by identifying the important features of each of these parent functions.

Linear Function: $y = x$

Sketch the graph

Domain	Range	x-intercept(s)	y-intercept(s)	Asymptotes (if any)

Quadratic Function: $y = x^2$

Sketch the graph

Domain	Range	x-intercept(s)	y-intercept(s)	Asymptotes (if any)

Cubic Function: $y = x^3$

Sketch the graph

Domain	Range	x-intercept(s)	y-intercept(s)	Asymptotes (if any)

Reciprocal Function: $y = \dfrac{1}{x}$

Sketch the graph

Domain	Range	x-intercept(s)	y-intercept(s)	Asymptotes (if any)

Square Root Function: $y = \sqrt{x}$

Sketch the graph

Domain	Range	x-intercept(s)	y-intercept(s)	Asymptotes (if any)

Cube Root Function: $y = \sqrt[3]{x}$

Sketch the graph

Domain	Range	x-intercept(s)	y-intercept(s)	Asymptotes (if any)

Absolute Value Function: $y = |x|$

Sketch the graph

Domain	Range	x-intercept(s)	y-intercept(s)	Asymptotes (if any)

Natural Exponential Function: $y = e^x$

Sketch the graph

Domain	Range	x-intercept(s)	y-intercept(s)	Asymptotes (if any)

Natural Logarithmic Function: $y = \ln x$

Sketch the graph

Domain	Range	x-intercept(s)	y-intercept(s)	Asymptotes (if any)

Sine Function: $y = \sin x$ on $-2\pi \leq x \leq 2\pi$

Sketch the graph

Domain	Range	x-intercept(s)	y-intercept(s)	Asymptotes (if any)

Cosine Function: $y = \cos x$ on $-2\pi \leq x \leq 2\pi$

Sketch the graph

Domain	Range	x-intercept(s)	y-intercept(s)	Asymptotes (if any)

Tangent Function: $y = \tan x$ on $-2\pi \leq x \leq 2\pi$

Sketch the graph

Domain	Range	x-intercept(s)	y-intercept(s)	Asymptotes (if any)

Problem Set

Complete the following problems outlined below. You must show all your work and/or provide an explanation to justify your answers.

1. Given that $f(x) = 2x - 3$, find the value of $f(x + h)$.

2. Given $f(x) = \frac{1}{x+2}$, find the value $\frac{f(x+h)-f(x)}{h}$

3. Find the domain of the function: $f(x) = \sqrt{x^2 - 4}$

4. Find the domain and range of the function: $g(x) = \sqrt{x - 2} + 1$

5. Find the domain of the function: $f(x) = \frac{7x}{x^2-36}$

6. Find the equation of the line parallel to the line $4x + 3y = 8$ and that passes through the point $(2, -1)$.

7. Find the equation of the line that has slope of $\frac{1}{2}$ and passes through the point $(-3,6)$.

8. Given the two lines $2x + y = 5$ and $y = \frac{1}{2}x + 7$ determine if the lines are parallel, perpendicular or neither. Justify your answer

9. If the point with coordinates $(3, k)$ is on the line $2x - 5y = 8$, determine the value of k.

For problems 10 through 13, use the given functions to determine the following compositions:

$$f(x) = 3x + 5 \text{ and } g(x) = x^2$$

10. $(f \circ g)(x)$

11. $(g \circ f)(x)$

12. $(f \circ f)(x)$

13. $(f \circ g)(-2)$

For problems 14 and 15, find the inverse of the function.

14. $f(x) = \frac{1}{3}x$

15. $g(x) = \frac{x-1}{5}$

For problems 16 and 17, verify the functions are inverses.

16. $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x}$

17. $f(x) = 3 - 4x$ and $g(x) = \frac{3-x}{4}$

18. Find the vertex of the function: $f(x) = (x+4)^2 + 3$

19. Given the function $f(x) = 3x^2 + 2x - 1$, determine if the function has a maximum or a minimum. **Justify your answer.**

20. Find the equation of the quadratic function with a vertex of $\left(0, \frac{1}{2}\right)$ and that passes through the point $\left(3, \frac{55}{3}\right)$ and opens upward.

21. Find the zeros of the function: $f(x) = x^4 - 3x^3$. **Make sure to identify any multiple roots.**

22. Given the function: $f(x) = (2x - 1)(x^2 + 1)(x - 5)^2$, identify the number of unique real roots.

23. Identify the x-intercept(s) of the function $f(x) = 4x^4 - 7x^3 - 2x^2$.

For problems 24 through 27, identify the vertical and horizontal asymptotes, if any, for each function:

24. $f(x) = \frac{5x}{x-1}$

25. $g(x) = \frac{2x^2}{x^2-9}$

26. $h(x) = \frac{2}{x-6}$

27. $k(x) = \frac{7x^4+2}{3x^2-2x-1}$

For problems 28 and 29, evaluate the functions for the indicated values. **Keep answers simplified that are written as exact values with positive exponents.**

28. $g(x) = 4^x + 1$

x	-2	-1	0	1	2
$g(x)$					

29. $k(x) = 2e^{-x}$

x	-2	-1	0	1	2
$k(x)$					

For problems 30 through 32, **expand** each logarithm completely.

30. $\ln \frac{(3x-5)^2}{7}$

31. $\log \frac{1}{z^3}$

32. $\ln \frac{x^4 y^2}{z^5}$

For problems 33 through 35, **condense** each logarithm to a single quantity.

33. $\log x - 2 \log y + 3 \log z$

34. $2 \ln 8 + 5 \ln(x - 4)$

35. $3 \ln 2 - 7 \ln x$

For problems 36 through 39, solve the following equations. **Make sure to note any extraneous solutions, if any.** quantity.

36. $8^x = 4$

37. $\ln x - \ln 8 = 0$

38. $2e^{3x} = 32$

39. $\ln x - \ln(x + 1) = 5$

40. Given the angle $\theta = \frac{4\pi}{3}$, identify the quadrant where this angle lies **and** evaluate $\sin\theta$, $\cos\theta$, and $\tan\theta$.

41. Given the $\sin\theta = \frac{3}{4}$, sketch a triangle and find the values of the other five trigonometric functions.

For problems 42 through 44, use your knowledge of the unit circle to evaluate the following functions.

42. $cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

43. $tan^{-1}(1)$

44. $sin^{-1}\left(-\frac{1}{2}\right)$

For problems 45 and 46, identify the amplitude and period for each function.

45. $y = \frac{3}{2}\sin\frac{\pi}{3}x$

46. $y = -10\cos 4x$

For problems 47 and 48, simplify the trigonometric expression.

47. $\sin x \sec x$

48. $\dfrac{1-\sin^2 x}{\csc^2 x - 1}$

For problems 49 and 50, solve the trigonometric equations over the interval $[0, 2\pi)$ using trigonometric identities.

49. $2\sin^2 x + \sin x - 1 = 0$

50. $2 \cos x + \sqrt{3} = 0$

Rubric

Criteria	Level 4 – 30 points	Level 3 – 24 points	Level 2 – 21 points	Level 1 – 15 points	No evidence 0 points
Knowledge of Mathematical Prerequisites	Demonstrates an **accurate and thorough** understanding of the prerequisite concepts. Executes calculations **completely and correctly** in order to demonstrate **mastery** of skills.	Demonstrates an accurate understanding of the prerequisite concepts. Executes calculations correctly in order to demonstrate skills.	Demonstrates minimal understanding of prerequisite concepts. Executes calculations but provides little evidence to demonstrate skills.	Demonstrates lack of understanding of prerequisite concepts. Lacks evidence to demonstrate skills.	Does not complete assignment.
Problem Solving Skills	Identifies **all** important elements of the problems and **accurately applies** appropriate concepts and skills. **Thoroughly** applies appropriate and logical strategies to provide **clear** evidence of **complete** solutions.	Identifies most important elements of the problems and applies appropriate concepts and skills. Applies appropriate strategies to provide some evidence of solutions.	Identifies some important elements of the problems but does not apply appropriate concepts and skills. Applies some strategies, but evidence of solutions is unclear.	Lacks important elements of the problems and does not apply appropriate concepts, skills, or strategies. Lacks evidence of solutions.	Does not complete assignment.